P004 INVESTIGATIONS OF ELASTIC WAVES PROPAGATION IN MODELS OF SANDS SEDIMENTS

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Summary.
We present a new approach to describe physical deforming processes in microinhomogeneous media in general, in particularly, sands sediments. Our approach is based on examination of individual interactions between particles. After it we used a new method of averaging of field forces for chaotic orientation of grains. The theoretical results are: velocity of P and S waves depends not only on average elastic modules of grains and porosity, but also on integral geometric parameters (average number of contacts, average size of a grain). Besides of it we create a new mechanism of wave attenuation. This mechanism produces a linear law of attenuation with frequency. Experimental results of authors have a satisfactory correspondence with theoretical predictions. There is a theoretical prediction about non-possibility of shock waves existence in media mentioned above.

Introduction.
In previous papers [1,2,3] authors used either phenomenological approaches or contact mechanism with strictly organizing system of grains. But only chaotic system produces an isotropic body. This problem has is interesting for geophysical exploration and specially interesting for excitation of seismic waves by explosions. On the other hand the well-known theory of mixtures is not useful for media, contain components with large jumps of mechanical properties between phases, if their concentrations are not very small [1]. Thus, it is necessary to examine a deforming process on the microscale range and after it to average the microscale field for creating a continued model.

Theoretical part.
Let's examine a two-dimensional situation, and imagine a medium as a set of chaotic orientated mesostructures (Fig1).

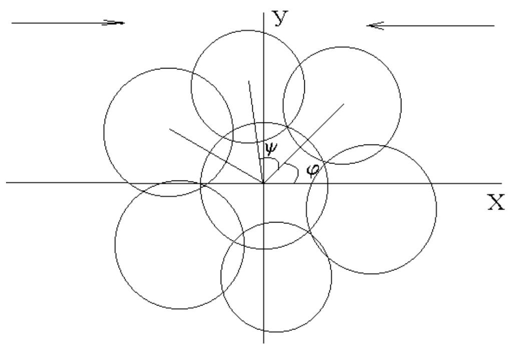

Fig.1.

We can write forces, acting on the grain, and make long-wave approximation ($d/\lambda<<1$) (d-diameter of grain, λ-wavelength) using usual Hertz law:

$$F=\frac{2E}{3(1-\nu^2)}\left(\frac{R1R2}{R1+R2}\right)^{1/2}\left(\{R1+R2\}-\{x_2-x_1\}\right)^{3/2} \qquad (1)$$

where: F – force of pressure, E – module of Young of material, $R1$, $R2$ - radiuses, ν - Poisson ratio, x_1 and x_2 – coordinates of center of mass of grains. The system is loaded by constant force, which provides initial rapprochement of grains on δ_0. The mesostructure consists of N+1 particles, which are contacted to each other. One of them is situated in the center, and the other N particles are its neighbors. The centers of mass of neighboring grains form vertexes of equilateral polygon with central angle ψ. By rotation on this angle the symmetry of the structure is remained (N – average number of contacts). These polygons are orientated in medium at random angels φ with axis X. Let's average the force acting on random grain by integrating on φ from 0 to ψ. In order to do this let's write equation of motion for X and Y components of displacement (U and V) depending on angle φ:

$U_{i\,tt}= A\{\delta_0+\Delta U_{i1}\cos\varphi+\Delta V_{i1}\sin\varphi\}^{3/2}\cos\varphi+A\{\delta_0+\Delta U_{i1+1}\cos(\varphi+\psi)+\Delta V_{i1}\sin(\varphi+\psi)\}^{3/2}\cos(\varphi+\psi)+ ...$
$+A\{\delta_0+\Delta U_{iN}\cos(\varphi+(N-1)\psi)+\Delta V_{iN}\sin(\varphi+(N-1)\psi)\}^{3/2}\cos(\varphi+(N-1)\psi);$ (2)

$V_{itt}= A\{\delta_0+\Delta U_{i1}\cos\varphi+\Delta V_{i1}\sin\varphi\}^{3/2}\sin\varphi+$
$+ A\{\delta_0+\Delta U_{i1+1}\cos(\varphi+\psi)+\Delta V_{i1}\sin(\varphi+\psi)\}^{3/2}\sin(\varphi+\psi)+ ...$
$+A\{\delta_0+\Delta U_{iN}\cos(\varphi+(N-1)\psi)+\Delta V_{iN}\sin(\varphi+(N-1)\psi)\}^{3/2}\sin(\varphi+(N-1)\psi);$ (3)

where $A=2E(2R)^{-2.5}/\{\pi\rho(1-\nu^2)\}$ if R1=R2=R, ΔU_{iN} and ΔV_{iN} – differences of displacements between the central grain and N-th neighbor grain for components U and V accordingly. Let's make expansion of Taylor expressions taking into account that $\delta_0>>U$, V. If R1≠R2 we can average the value A by integrating with respect to R form R1 to R2.

Let's introduce the friction force, considering that it is directed on tangent to the surface of contact and it is proportional to the force, which approach spheres. In order to separate the nonlinear effect we can limit to the first term of expansion. This give a nonlinear term in long-wave approximation of the order of U^2/λ^2, while the second term of expansion gives effect like RU^2/λ^3, and this value is of more higher order than the previous term. In according to long-wave approximation (d-diameter of grain) for n -th neighbor (n=1,2,...N):

$\Delta(U,V)_{in}= d\cos(\varphi+(n-1)\psi)(U,V)_x + d\sin(\varphi+(n-1)\psi)(U,V)_y+$
$+d^2/2(\cos^2(\varphi+(n-1)\psi)(U,V)_{xx}+ \sin^2(\varphi+(n-1)\psi)(U,V)_{yy}+$
$+2\sin(\varphi+(n-1)\psi)\cos(\varphi+(N-1)\psi)(U,V)_{xy}+...$ (4)

Changing of angle between the grains during deformation process is:

$\Delta(\varphi+(n-1)\psi)=(-\Delta U_{i(n-1)}\cos(\varphi+(n-1)\psi)+\Delta V_{i(n-1)}\sin(\varphi+(n-1)\psi)/d$ (5)

For estimation of average displacement field taking into account friction and nonlinear term it is necessary to average forces on each contact and determine their sum:

$$1/\psi\left[\int_0^{\psi} F(\varphi)d\varphi+\int_0^{\psi} F(\varphi+\psi)d\varphi+...+\int_0^{\psi} F(\varphi+(N-1)\psi)d\varphi\right]=\frac{N}{2\pi}\int_0^{2\pi} F(\varphi)d\varphi \qquad (6)$$

Linear approximation:

$U_{tt}=9EN\delta^{1/2}{}_0(d)^{-0.5}/\{32\pi^2\rho(1-\nu^2)\}\{U_{xx}+1/3U_{yy}+2/3V_{xy}\pm k_f(V_{yy}+1/3V_{xx}+2/3U_{xy})\}$ (7)

$V_{tt}=9EN\delta^{1/2}{}_0(d)^{-0.5}/\{32\pi^2\rho(1-\nu^2)\}\{V_{yy}+1/3V_{xx}+2/3U_{xy} \pm k_f(U_{xx}+1/3U_{yy}+2/3V_{xy})\}$
(8)

where k_f – friction of grain material.

Hence, velocity of P-waves in such media is:

$C_p=(3/4\pi)(EN/2\rho(1-\nu^2))^{1/2}(\delta_0/d)^{1/4}$ (9)

Expressing δ_0 by F_0 we can write:

$C_p=(3/4\pi)(N(1-f)/2\rho)^{1/2}(E/(1-\nu^2)d)^{1/3}(3F_0)^{1/6}$ (10)

F_0 is force acting on the single particle. There is a possibility to express the force, acting on the medium F_1 by the force acting on a single particle.

$F_1=F_0 l\sqrt{2}/d$, where l is size of medium.

$$C_p=2^{1/12}(3/4\pi)(N(1-f)/2\rho)^{1/2}(E/(1-\nu^2)d)^{1/6}(3F_1)^{1/6} \quad (11)$$

Dependence on porosity is given by expression $\rho_{medium}=\rho_{grain}(1-f)$, where f - porosity, so that $C_p\sim(1-f)^{1/2}$. As we see in linear approximation the sandy medium is described by one-constant theory of elasticity. C_p is proportional to the square root form average number of contacts and there is inverse proportional relation of it to 1/6-th power from radius of a grain. Now we may use formulas of theory of mixtures to determine parameters of a real media (to include the effect of liquid saturation) because the difference between phases is not so large. For plane waves ($\frac{\partial}{\partial y}=0$) we have more simple equation of motion, neglecting the quadratic terms:

$$U_{tt}=K\{U_{xx}\pm k_f V_{xx}/3\} \quad (12)$$

$$V_{tt}=K\{V_{xx}/3 \pm k_f U_{xx}\} \quad (13)$$

where $K=9EN\delta^{1/2}{}_0(d)^{-0.5}/\{32\pi^2\rho(1-\nu^2)\}$. The formulas (12) and (13) show that attenuation is proportional to the first power of frequency. The second result is connection between U and V components and cross-radiation between components. Equations with quadratic terms are:

$$U_{tt}=K\{U_{xx}\pm k_{\text{тр}}V_{xx}/3\pm(U_x^2+4V_x^2+2U_xV_x)/4d\} \quad (14)$$

$$V_{tt}=K\{V_{xx}/3 \pm k_{\text{тр}}U_{xx}\pm(V_x^2+4U_x^2+2U_xV_x)/4d\} \quad (15)$$

The nonlinear terms in (14,15) have a special structure. This structure causes the extension of initial pulse. It means that the shock waves in such media are absent.

Experimental results.

On a special constructed installation (Fig.2) experiments of wave velocities measuring as a function of average diameter of a grain were made. Experiments were carried out with fractions of nearly equal size of grains.

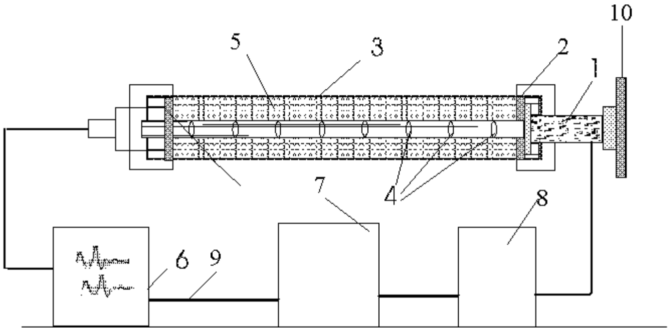

Fig.2. The scheme of installation for measurement of P-waves velocities in sands media. 1 – pulse source, 2 – washers, 3 – steel cylinder, 4 – sensors, 5 – sandy medium, 6 – recorder, 7 – generator of pulse oscillations, 8 – managing module, 9 – connection lines, 10 – support.

These data are presented on the Fig.3.

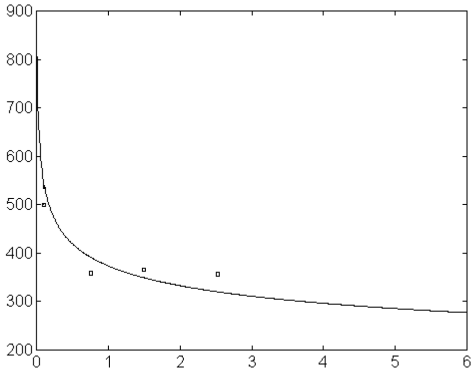

Fig.3. On the x-axis is average diameter of grains (mm). On the y-axis is velocity of P-waves (m/s). Points are experimental data while curve is a theoretical relation.

We can conclude that there is a satisfactorily agreement between theoretical prediction and experimental results.

Conclusion.

1. A new method of microinhomogeneous media discription if offered. This method takes into account average diameter of a grain, average number of contacts, porosity, mechanical properties of grains and their chaotic orientation.
2. Experimantal measurements show a satisfactory agreement with theoretical predictions for relation between wave velocity and diameter of a grain.
3. In this paper it is shown that the front of discontinuty of state paramemers in grain-structured media is unstable. This leads to transformation of shock waves into continuous pulse of compression.

References.

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